

## Birthday Experiment Assignment Answer key

**Due Date:** \_\_\_The same as homework 2\_\_\_\_\_

**Assignment Goal:** To understand the computation of the probability that at least two people in a group of  $n$  people share the same birthday and the probability that nobody in a group of  $n$  people share the same birthday.

To understand the idea of those probabilities try the Birthday Experiment applet at the following website:

[http://socr.stat.ucla.edu/htmls/SOCR\\_Experiments.html](http://socr.stat.ucla.edu/htmls/SOCR_Experiments.html)

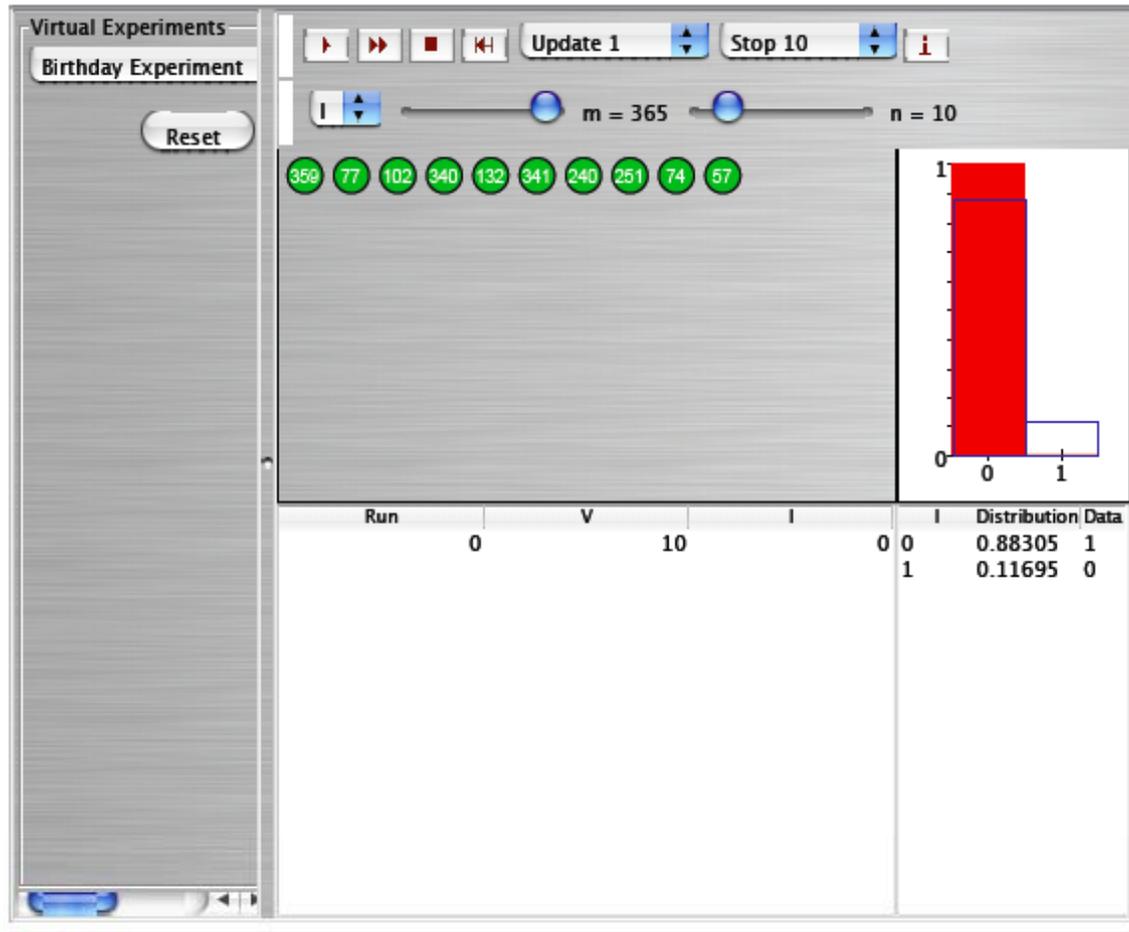
Select the Birthday Experiment from the drop-down list on the top-left.

- (a) Draw 10 people at random from the population and observe their birthdays. This is like drawing 10 balls ( $n=10$ ) at random with replacement from an urn containing 365 balls ( $m=365$ ). To do that, set  $n=10$ , set  $m$  to 365. Click on the run-one-step button  $\Rightarrow$

How many distinct balls did you get? \_\_\_\_\_10\_\_\_\_\_ Is that the same as the value of  $V$ ? \_\_Yes\_\_\_ What is the value of  $V$ ? \_\_\_\_\_10\_\_\_\_\_

Were there any balls that repeated? (i.e., any red ball in your set?) \_\_\_No\_\_\_\_\_ If there are two balls with the same number,  $I$  should be 1. If not,  $I$  should be 0. What is your  $I$ ? \_\_\_\_\_0\_\_\_\_\_

Attach a printout of your applet with your work.



(b) In (a) we did only one run of the experiment. That won't take us too far. We need to repeat the same experiment many times to see how often we get 10 green balls and how often we get at least a red ball. Keep  $n=10$  and  $m=365$  and set update=1 and stop=10. Click on the run-multiple-steps button . You will only see that last run of the experiment in the coins drawn, but you will see all 10 runs and the V's and I's and the distribution of the I's.

Was there a red ball in the last run? (The last one is the one shown on the bottom of the screen)\_No\_\_

In how many runs was the number of distinct balls equal to 10? \_\_\_\_\_9\_\_\_\_\_

In how many runs was V equal to 10? \_\_\_\_\_9\_\_\_\_\_

In how many runs was I equal to 0? \_\_\_\_\_9\_\_\_\_\_

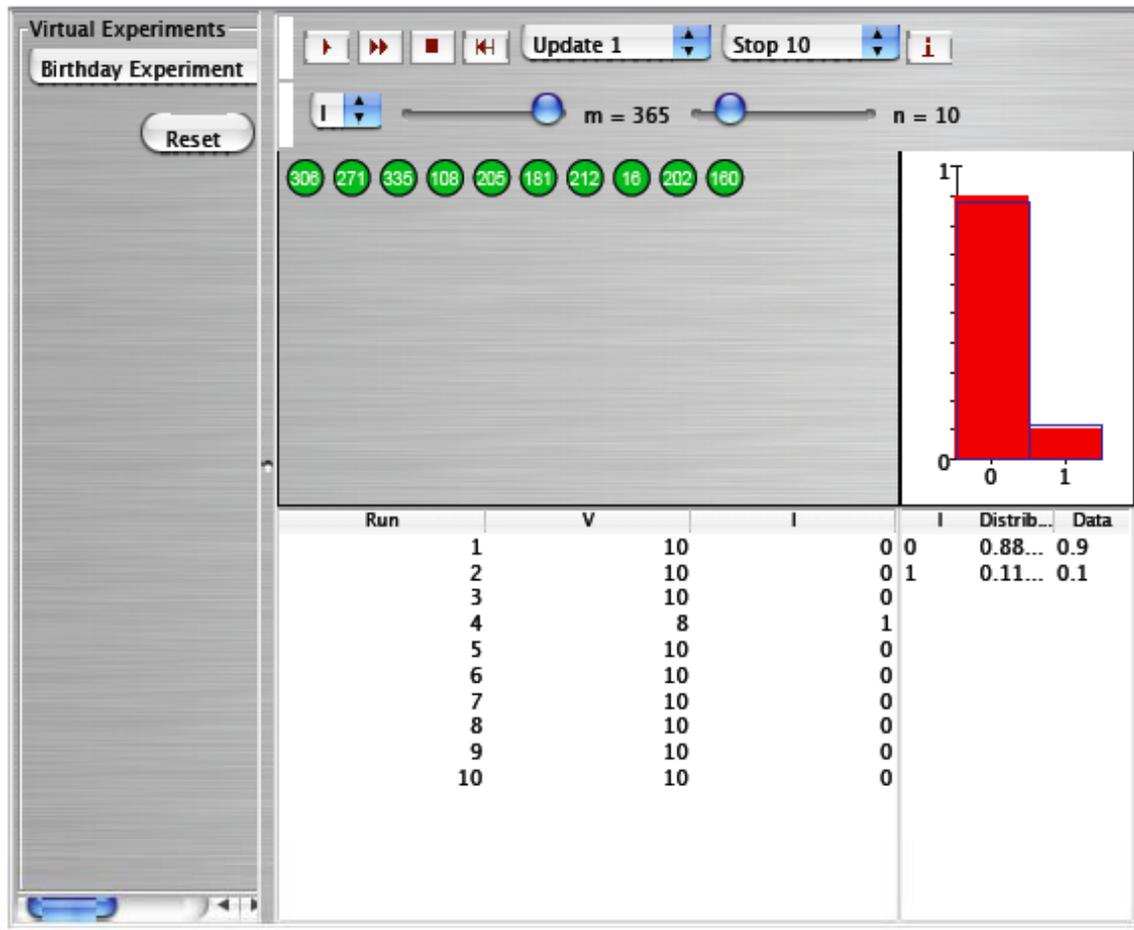
In how many runs was I equal to 1? \_\_\_\_\_1\_\_\_\_\_

What is the theoretical distribution of I on the top right hand side giving you? (**Notice, that if you don't see the distribution of the I, you can go up to the left hand corner, near the m , and select I instead of V**)  $P(I=1)=0.88305$  and  $P(I=0)=0.11695$

According to your 10 runs, the probability that no two people in a group of 10 share the same birthday is?  $0.9$

The probability that at least two people in a group of 10 share the same birthday is, according to your 10 runs  $0.1$

Attach a printout/snapshot of the applet with your results.



(c) With 10 runs, you have not got too close to the true probability. So what about trying 10000 runs of the experiment? Don't even dream of seeing every single one of these. So, reset the screen, and set the stop at 10000 and the update at 1000. This means you will only see every 1000 run. But the final distribution with all 10000 runs will appear on the right.

Look at the distribution of the I on the right. What proportion of times did the run have at least a red ball? \_\_\_\_\_ 0.1224 \_\_\_\_\_

This is the approximation to the probability (looking at the data distribution) that at least two people in a group of 10 share the same birthday. The true probability is? 0.11695

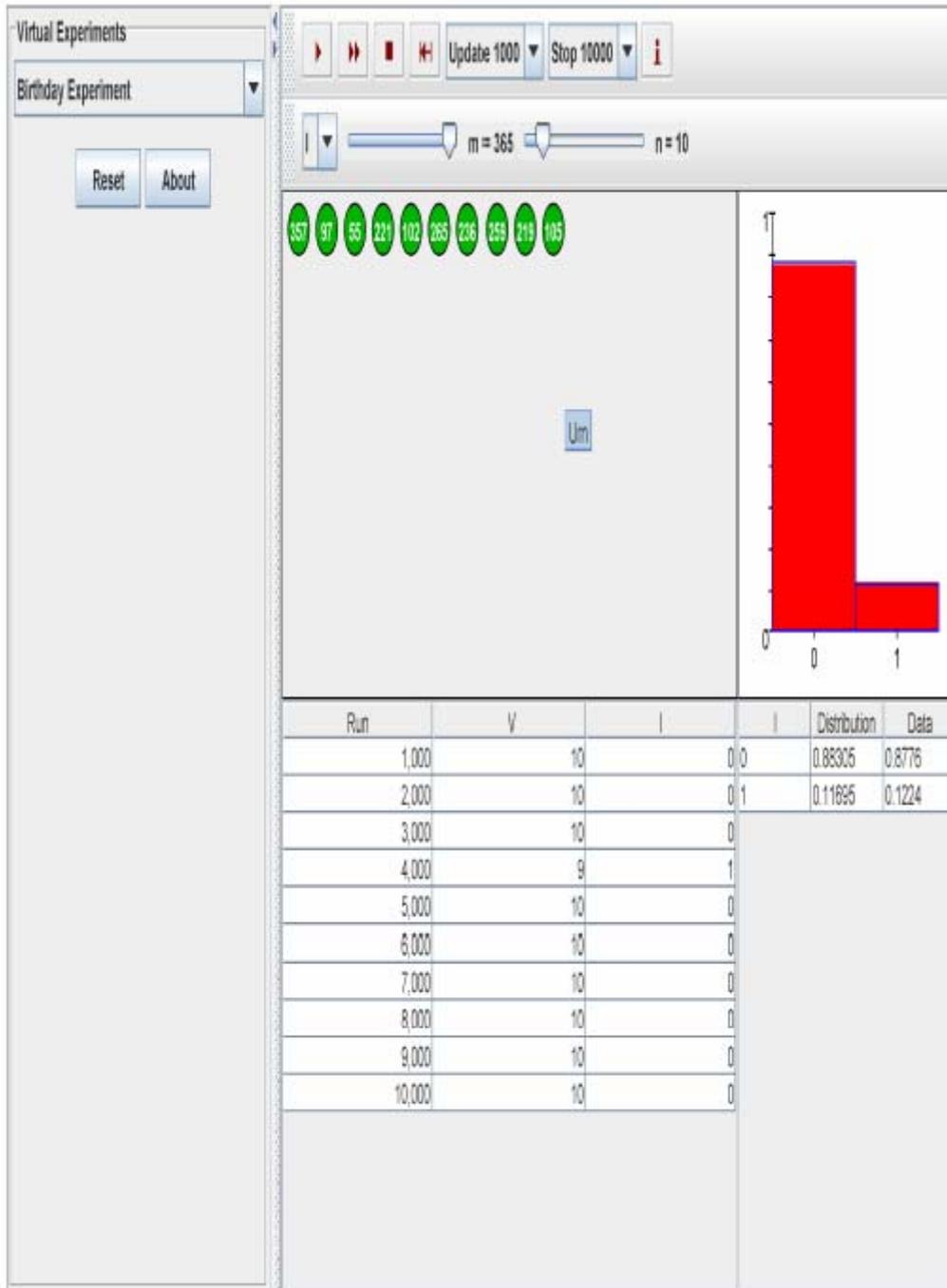
What proportion of the 10000 experiments had all 10 balls be distinct?  
\_\_\_\_\_ 0.8776 \_\_\_\_\_

This is the approximation to the probability that nobody in a group of 10 share the same birthday.

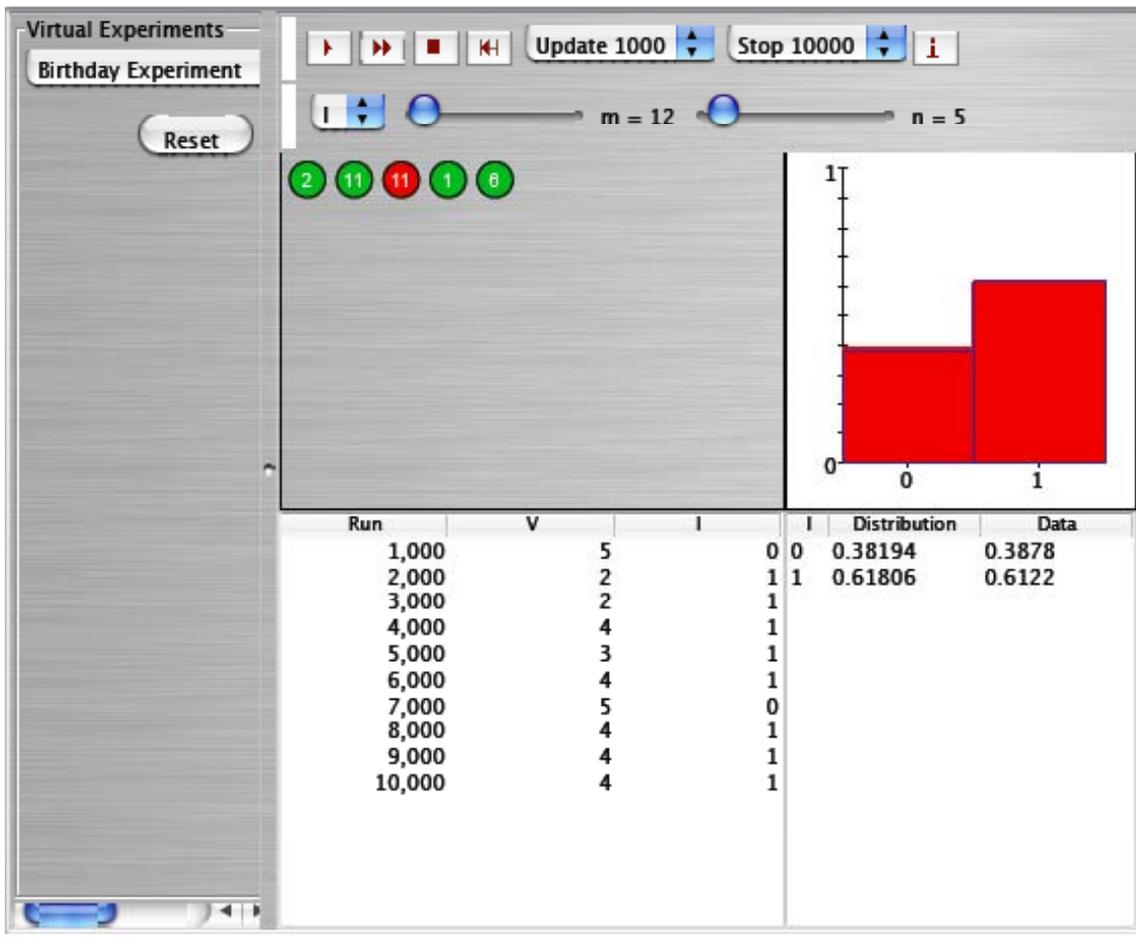
What is the theoretical probability \_\_\_\_\_ 0.88305 \_\_\_\_\_

How is the first probability related to the one you calculated in (b) using only 10 experiments ?

Attach a printout/snapshot of the applet at the end of the 10000 runs.



(d) Use what you have learned above to determine empirically the probability that at least 2 people in a group of 5 share the same **birthmonth**. Determine also the probability that nobody in a group of 5 share the same month. Write your answer here and attach a printout/snapshot of the applet with the final runs.  $P(I=1) = 0.38194$ , that is the empirical probability that at least two people in a group of 5 share the same birthmonth.  $P(I=0) = 0.61806$ , that is the empirical probability that nobody in a group of 5 share the same birthmonth.



(e) Use what you have learned above to determine empirically how large should the group of people observed be for the probabilities of at least two sharing the same birthdays and the probability of nobody sharing same birthday to be 50%-50%. This may take some trial and error. Turn in your final answer written here and attach your applet. Use for this the empirical distribution (in red) and the theoretical distribution (in blue).

When  $n=23$ , theoretically,  $P(I=0)=0.5073$  and  $P(I=1)=0.4927$ .

